Understanding S parameters in time domain and the application to Two X Automatic Fixture Removal of high speed interconnects

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Abstract

Although most engineers are trained to think and reason in both the time and frequency domains, Scattering (S) parameters are most often considered as frequency domain entities with little reference to their time domain picture. In this paper, it is shown that S parameters as represented in the time domain offer a complementary perspective that unfolds effects that appear combined in the frequency domain, thanks to the uncertainty

principle, a fundamental property of the time and frequency domains representations. To enable the discussion, we step back to highlight the physical meaning of the S parameters and the role of the reference impedance. It is emphasized that S parameters are physical quantities independent of whether they are represented in time or frequency. We present the behavior of the S parameters in the time domain for simple discontinuities that appear in a typical high speed interconnect (pad capacitance, via inductance, etc.). One important application where the time domain picture emerges is time domain reflectometry (TDR). Therefore, we show how the TDR or instantaneous impedance can be calculated from the S parameters and shade some light on the limitations of the frequency domain data that may affect the calculation accuracy. Finally, the 2X Automatic Fixture Removal method is explored, where the pivotal role of the time and frequency pictures becomes evident. I hope the discussion, examples and applications will motivate engineers to develop their understanding and exploit the time domain representation to debug complex systems and create novel fixture extraction algorithms that suit their needs.

Author(s) Biography

Sameh Elnaggar has been a Signal Integrity Engineer with Semtech since 2017. Prior to that, he was a Postdoc at the School of Engineering and Information Technology, University of New South Wales, researching the properties of nonlinear metamaterials. Throughout his career, he held several engineering jobs that spans automation, optics, power systems, RF and signal integrity. He holds a Ph.D degree in Electrical and Computer Engineering from the University of New Brunswick and is a registered Professional Engineer (P.Eng).

Introduction

For every Signal Integrity (SI) Engineer, S parameters represent the de-facto interconnect model for many reasons. Firstly, they are measured directly via a Vector Network Analyzer (VNA), one of the most precise high frequency equipment. Secondly, they conceal the implementation, thus making their exchange between design team members and with customers and vendors convenient. Thirdly, most commercial Full wave simulators and EDA tools either produce or accept S parameters files (commonly known as Touchstone, or SnP files).

Despite the ubiquitous use of S parameters in both industry and academia, they come with their own subtlety. For example to many engineers, the role of the reference impedance and impedance re-normalization are unclear. The role of impedance can be particularly confusing to engineers who design and characterize systems that operate in a host environment with an impedance different from 50 Ohm. This is especially true for designers of video systems and those involved in the design of packages and PCBs with interconnect impedances other than the conventional 50 Ohm.

One of the purposes of this paper is to re-introduce S parameters to SI engineers in a tangible way that establishes the connection with the physics of the system (i.e., circuit or interconnect). To reach this goal, we briefly discuss in the next section linear time invariant (LTI) systems and how their behaviors are fully described by the impulse response or equivalently the transfer function. We then extend the discussion to multiport networks (Single ended and differential traces are concrete examples). This generic approach leads in a natural way to a better appreciation of how the system behavior transcends the representation, whether it is in time or frequency. To make this pivotal concept clear, we draw an analogy between transformation of 2D vectors and LTI systems.

Equipped with the conceptual description of a LTI system, the S parameters are defined in such a way that they represent the network intrinsic properties. The meaning of the reference impedance and impedance normalization follow naturally. Furthermore, we show how the S parameters are represented in the time domain.

To better illustrate the physical meaning of the S parameters, Return and Insertion losses are calculated in both the time and frequency domains for simple lumped elements. Additionally, the intimate connection between S parameters in time and the instantaneous impedance, commonly known as the "TDR" is highlighted.

Besides the computation of instantaneous impedance (i.e., TDR), the time and frequency representations of the S parameters find applications in fixtures extraction. Hence, in this paper we show how the combination of both the time and frequency representations enables the extraction of interconnects' S parameters. Although there are various time-frequency fixture extraction methods, we limit the presentation to the elegant 2X-

Automatic Fixture Removal (2X-AFR) method. Readers interested in other techniques can refer to Ref. [1].

Physical meaning of S parameters

Fundamentally, the physics of most electrical systems is captured by a set of linear equations. Indeed, the relations between voltage and current for the building blocks of circuits (resistors, capacitors and inductors) are described by linear algebraic or differential equations. Deeper under the surface, the behavior of all electric systems obeys Maxwell's equations that are nothing but a set of four differential "linear" equations. Furthermore, circuit elements and materials are time invariant, meaning that the parameters (resistance, capacitance, inductance, permittivity, permeability, etc.) do not change with time. This implies that electric systems usually belong to the class of LTI systems.

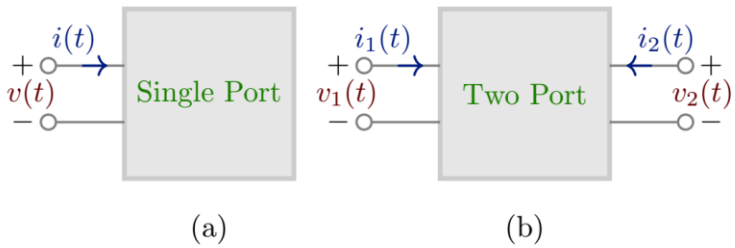

Figure 1. Examples of multiport networks.

Fortunately, LTI systems have general properties that make their description relatively easy. To show this, consider the single port in Figure 1(a). The network can represent any combination of circuit elements (capacitors, inductors, microstrips, vias, etc.). As is already known, from the port perspective the network behavior reduces to the relation between the terminal voltage $v(t)$ and current $i(t)$. Linearity of the network implies that $v(t)$ and $i(t)$ are linearly related. For instance if $i(t)$ is amplified five times, $v(t)$ is amplified by the same factor. Additionally, the voltage $v(t)$ due to an input current source $i(t) = i_1(t) + i_2(t)$ is the sum of the voltages due to $i_1(t)$ and $i_2(t)$ separately. Therefore, from a terminal voltage and current perspective, the network (or generally a system) linearly maps the current waveform to a voltage waveform. It is crucial to note that this mapping is a property of the system not the input.

To reveal the nature of the mapping between $v(t)$ and $i(t)$, we refer to the simplified model in Figure 2. $v(t)$ and $i(t)$ are considered as vectors in the plane. The mapping from $i(t)$ to $v(t)$ is represented by the 2×2 matrix Z. For a current vector $i(t)$ in the plane (or equivalently a given current waveform), the matrix Z transforms $i(t)$ into $v(t)$. It is well known that for a wide class of matrices there is some coordinate system (e_1, e_2) that permits the operation of Z to be simplified. Indeed, any vector pointing in the e_1 or e_2 directions will be scaled but still point in the same direction. For example, the e_1 component of $i(t)$ is compressed while the e_2 component is elongated. These directions are known as the eigenvectors of Z. They are characteristic directions of the matrix Z. The eigenvalues of Z quantify the scales along e_1 and e_2.

Keeping the above analogy in mind, the characteristic directions of a LTI system are the sinusoids. Indeed if the input to a circuit is a sinusoid, the output is still a sinusoid, albeit a change in magnitude and phase. Mathematically, this change in magnitude and phase is represented by a complex number, which is the eigenvalue corresponding to the given sinusoid. Each sinusoid is identified by its frequency f. The eigenvalue $Z(f)$ at frequency f, is nothing but the system transfer function.

The above arguments clearly show that it is convenient to use the frequency domain when characterizing circuits. Of course, this does not come as a surprise to an electrical engineer. Loosely speaking, the frequency domain is equivalent to aligning the coordinates' axes in the LTI system preferred directions hence rendering the analysis much simpler. In this "coordinate system" the effect of Z reduces to mere number multiplications:

$$V(f) = Z(f) \times I(f).$$

Another more human friendly representation is time. Humans find it easier to reason and think in terms of sequences of events in the time domain. Although devices and interconnects may be characterized in the frequency domain, ultimately performance is evaluated in time. Consider for example a communication channel, where symbol streams are transmitted from one end to another. Eventually, performance is assessed using time domain parameters such as Symbol Error Rates, Channel Operating Margin (COM) … etc. If the coordinate system is "aligned" in the time coordinates (equivalently we choose the (x, y) coordinates in Figure 2), the network behavior reduces to the convolution operation:

$$v(t) = z(t) * i(t),$$

where $z(t)$ is the inverse Fourier transform of $Z(f)$.

The above discussion can be extended to multiport networks; for example the two port network shown in Figure 1. In frequency, the terminal voltages are related to the currents by the well known Z parameters

$$V_1(f) = Z_{11}(f)I_1(f) + Z_{12}(f)I_2(f)$$

and

$$V_2(f) = Z_{21}(f)I_1(f) + Z_{22}(f)I_2(f).$$

Or equivalently in time as

$$v_1(t) = z_{11}(t) * i_1(t) + z_{12}(t) * i_2(t)$$

and

$$v_2(t) = z_{21}(t) * i_1(t) + z_{22}(t) * i_2(t).$$

The relation between $i_1(t)$ and $i_2(t)$ from one side and $v_1(t)$ and $v_2(t)$ from the other, is a system property that does not depend on the inputs.

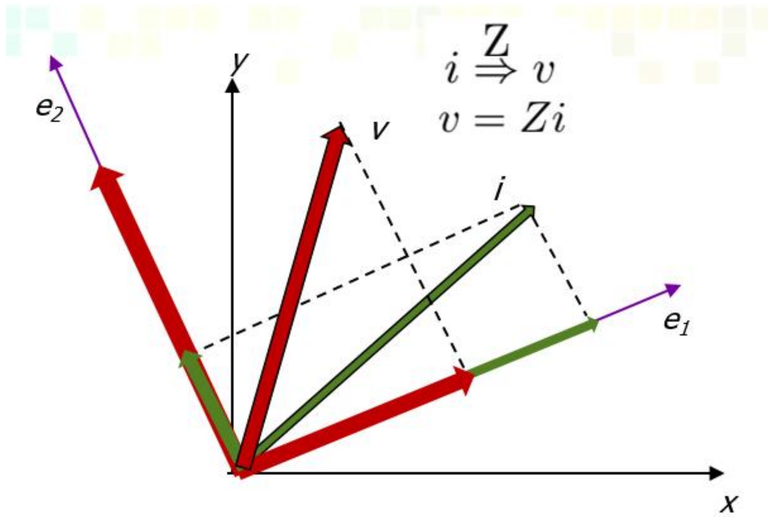

Figure 2. A simplified 2D model of the single port network.

S parameters

In this section, we show that the S parameters represent a system property. This has two important implications (1) they do not depend on the input and (2) they are, very much like the Z parameters, transcend their representation. Time and frequency domains emphasize complementary facets of the parameters.

From their name, S parameters describe an electric circuit using the strength of scattering and transmission of incident waves that impinge the system. Such description is clearly understandable when one characterizes microwave circuits. In this case, the inputs are usually waves propagating in waveguides that lead to the system. On the other hand, interconnects convey signals from a transmitter to a receiver. Usually the signals strength are defined by their peak or peak to peak voltages. Therefore, a description in terms of voltages and currents is convenient. Nevertheless, at high baud rates, communication systems resemble microwave circuits and it is practical to exploit the extensive toolset a microwave or antenna engineer uses. Accordingly, a SI engineer deals with the two aspects: (1) voltages and currents more common in circuit design, and (2) waves naturally appearing in microwave networks. Hence, it is important to assimilate the two perspectives.

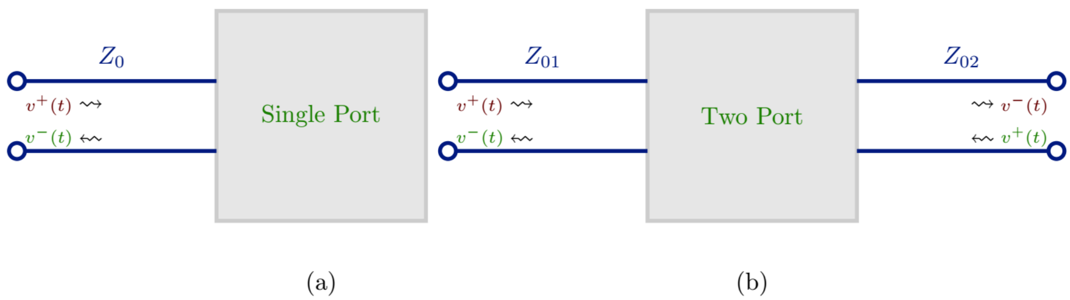

Figure 3 Electric network connected through its ports to a measuring instrument (VNA) via an ideal cable. (a) Single Port. (b) Two Port.

Unfortunately, many times S parameters are introduced as mere mathematical constructs that mimic waves appearing on guided structures. Clearly, as will be shown, S parameters represent real physical quantities that exist even at very low frequency and DC. Consider for example a Device Under Test (DUT) as the one shown in Figure 3(a) or an interconnect as in Figure 3(b). In both cases, we assume that the electric network is attached to a VNA via ideal cables. For the interconnect example, the cables on the left and right sides do not have to be the same, i.e, Z_{01} not necessarily equals to Z_{02}. Instead of focusing our attention of the voltage and current at the terminals (circuit design perspective), we consider the voltage and currents at the cables some distance l from the ports (microwave circuits perspective). As l becomes smaller (approaching the ports), v and i approach those at the terminals. It is well known that between the conductors of the cables the electric and magnetic fields appear as the superposition of the cable modes moving in the forward and backward directions [2]. Although a cable supports infinite number of modes, only the Transverse Electromagnetic (TEM) mode propagates at all frequencies. If the cable dimension is small enough, other modes are below their cut-off frequency and do not propagate [3] [4]. This means that the fields on the cable are reduced to those of the TEM mode only. Additionally, a TEM mode can be rigorously described by voltage and current quantities. This is not true for general waveguides that do not support TEM modes (for instance hollow waveguides). Moreover, the voltage and current of a wave travelling in some direction (forward '+' or backward '-') are related by a constant quantity: the cable characteristic impedance Z_0. The impedance depends on the cable internal structure and material. Accordingly, the voltage and current on the cable can be written down as

$$v = v_+ + v_-$$

and

$$i = \frac{v_+}{Z_0} - \frac{v_-}{Z_0}.$$

Note that the current of the backward wave is negative, emphasizing the fact that power is flowing in the negative direction (away from the network). In terms of v and i, the amplitudes of the waves can be written as

$$v_+ = \frac{v + Z_0 i}{2}$$

and

$$v_- = \frac{v - Z_0 i}{2}.$$

The network determines the relative strength of the forward and backward waves. To see this consider the single port network in Figure 3(a). At the port the impedance, a system parameter that can be represented in frequency and time, relates the voltage and current. Therefore,

$$V_-(f) = \Gamma(f) V_+(f),$$

where

$$\Gamma(f) = \frac{Z(f) - Z_0}{Z(f) + Z_0}$$

is the reflection coefficient. In the time domain, the relation takes the form

$$v_-(t) = \gamma(t) * v_+(t),$$

where $\gamma(t)$ is the reflection coefficient in the time domain and is equal to the inverse Fourier transform of $\Gamma(f)$.

In general, the S parameters represent the relation between the incident and scattered waves appearing on the transmission lines (TLs) attached to the network ports. They represent a system property much like the Z parameters, but with a caveat: they also depend on the transmission lines. Changing the TLs, changes the fraction of reflected to incident waves. However, the network must fix their sum. This means that regardless of the TLs impedances, the S parameters will always produce the same intrinsic network parameters ($Z, Y, ABCD\ etc.$). Moreover, knowing the S parameters for a given set of connecting TLs permits the calculation of the parameters for any set of TLs, which is nothing but impedance renormalization.

For a two port network the S parameters in the time domain take the form

$$v_{1-}(t) = s_{11}(t) * v_{1+}(t) + s_{12}(t) * v_{2+}(t)$$

and

$$v_{1-}(t) = s_{21}(t) * v_{1+}(t) + s_{22}(t) * v_{2+}(t),$$

where $s_{mn}(t)$ is the inverse Fourier transform of $S_{mn}(f)$. Physically $s_{mn}(t)$ is the waveform appearing at the m^{th} port due to an incident impulse wave that impinges the n^{th} port.

The strength of combining the time and frequency domains relies mainly on a fundamental property that links the two domains: the uncertainty principle. This is the same principle known in physics as the "Heisenberg uncertainty principle". In simple terms: localized events in time appear spread in frequency and vice versa. The widths of signals in time Δt and frequency Δf are constrained by the following inequality [5].

$$\Delta t \Delta f > \frac{1}{2\pi}.$$

The implication of such relation is far reaching. Resonance for example is a localized event in frequency. Therefore, it is expected that effect of resonance sustains over a long period of time. On the other hand, reflection from a lumped element embedded in a host transmission line is localized in time; hence, its effect appears over a wide frequency band. To have a deeper understanding of how S parameters look like in the time domain, scattering from simple discontinuities is considered in the next section.

S parameters for simple discontinuities

In this section, we calculate the S parameters in the time domain for simple elements. The calculation is based on time domain arguments only. The results themselves can be obtained from well-known frequency domain expressions that are derived and repeatedly used in microwave textbooks [3] [4]. However, the purpose of the discussion is to highlight the route to such expressions based on fundamental circuit properties. To fix ideas, consider the situation of a shunt capacitance C embedded in a trace of uniform characteristic impedance Z_0 (Figure 4). The configuration may represent a first order model of a pad or a connector that interfaces a coaxial cable and a PCB.

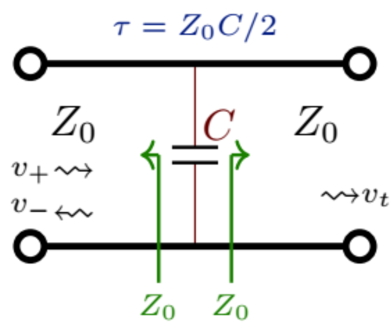

Figure 4 Capacitor embedded in a uniform trace.

To find the relation between v_-, v_t and v_+ , we apply Kirchhoff's laws at the discontinuity. Since voltage and current must be continuous,

$$v_+ + v_- = v_t$$

and

$$i_+ + i_- = i_c + i_t.$$

Next we use the circuit relations linking currents and voltages. Note that on the TL, the reference impedance relates the current and voltage to one another. Additionally,

$$i_c = C\frac{dv_c}{dt} = C\frac{d(v_+ + v_-)}{dt} = C\frac{dv_t}{dt}.$$

Combining the circuit relations with Kirchhoff's laws, it is straightforward to show that the incident and reflected waves on the line satisfy the following equation

$$\frac{dv_-}{dt} + \frac{1}{\tau}v_- = -\frac{dv_+}{dt},$$

where the time constant $\tau = Z_0C/2$. The above equation can be solved by standard methods (for example Laplace transform) to give

$$s_{11}(t) = -\delta(t) + \frac{1}{\tau}u(t)e^{-\frac{t}{\tau}},$$

where $\delta(t)$ and $u(t)$ are the standard delta and step functions [5] [6]. Note that $s_{11}(t)$ is the reflected wave when the capacitor is impinged by an impulse wave. The table shown in Figure 5 presents the S parameters for six different discontinuities.

Continuing with the shunt capacitance example, more insight is gained when the physics behind the different terms appearing in the expressions of $s_{11}(t)$ and $s_{21}(t)$ is revealed. Consider the time instant just before the incident impulse $v_+(t)$ reaches the discontinuity. At this instant, the voltage across the capacitor v_c is still zero. As the impulse reaches the capacitor, the capacitor resists the change in v_c and instantaneously acts as a low impedance. Hence, the wave flips and reflects back to the source. This is highlighted by the negative sign of the delta in $s_{11}(t)$. Additionally, the incident very high intensity current charges the capacitor. Later, C discharges through the resistance seen by its terminals, which is the parallel combination of the TLs Z_0 at the two sides of the discontinuity (Figure 4). Note that the cases shown in the fifth and sixth columns of Figure 5 are the dual of one another. This means that the situation where we have a series inductance is the analogue of the one discussed here. Scattering from the series inductance is then the decay of the magnetic field stored inside the inductance. The decay time constant is L divided by the equivalent resistance $2Z_0$. Note also that the inductor appears as a high impedance at the initial time, this explains why $v_-(t)$ has the same sign as $v_+(t)$.

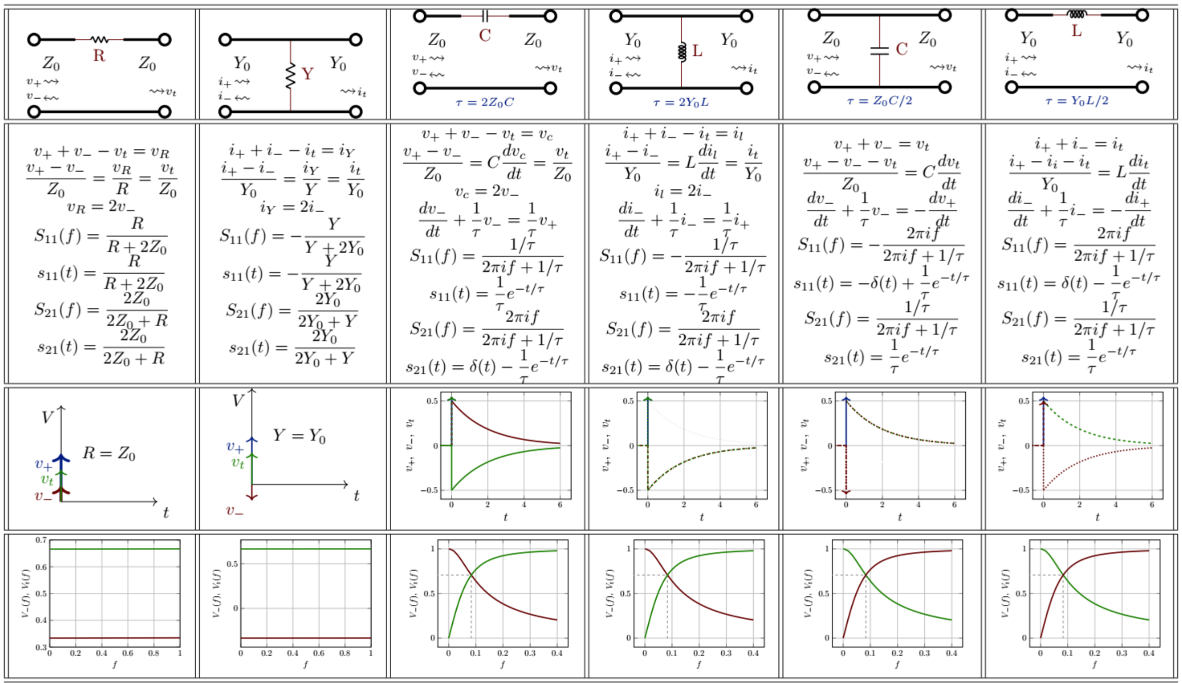

Figure 5 S parameters for simple discontinuities depicted in the first row. The second row present the main circuit relations. The third and fourth rows show the time and frequency response, respectively.

It is worth noting that in general the transmitted wave v_t leaving a discontinuity is not an impulse anymore. v_t carries with it a signature of the discontinuity (for instance the time constant τ). If v_t hits another discontinuity down the line, the reflected signal from the second discontinuity will combine the signatures of both discontinuities.

TDR and its relation to S parameters

Every SI engineer considers the instantaneous impedance $\hat{z}(t)$ as an indispensable parameter that unfolds the discontinuities in the time domain and provides a picture of the interconnect. Nevertheless for some engineers, the link between $\hat{z}(t)$ and the S parameters is somehow unclear for two main reasons. Firstly, the S parameters are usually considered frequency domain objects, while $\hat{z}(t)$ is a time domain quantity. Secondly, it is not clear how the frequency domain data affects the calculation of $\hat{z}(t)$. In this section, we briefly address these two points.

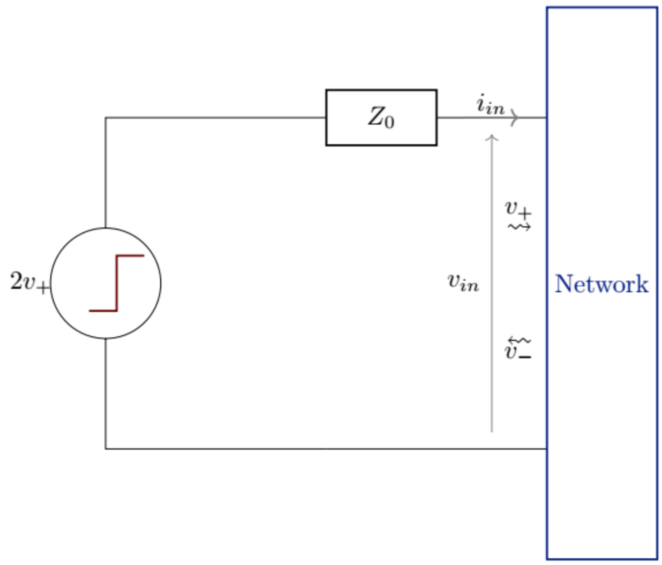

Figure 6 Setup used to define the instantaneous impedance.

Referring to Figure 6, the instantaneous impedance $\hat{z}(t)$ is defined to be

$$\hat{z}(t) = \frac{v_{in}(t)}{i_{in}(t)}$$

for a given source $2v_+(t)$. In terms of the incident and reflected waves

$$\hat{z}(t) = Z_0 \frac{v_+(t) + v_-(t)}{v_+(t) - v_-(t)}.$$

Noting that $v_-(t) = s_{11}(t) * v_+(t)$ and $\delta(t) * v_+(t) = v_+(t)$, the instantaneous impedance can be written in terms of the S parameters

$$\hat{z}(t) = Z_0 \frac{[\delta(t) + s_{11}(t)] * v_+(t)}{[\delta(t) - s_{11}(t)] * v_+(t)}.$$

The above expression shows how $\hat{z}(t)$ can be calculated in terms of the S parameters for any input waveform $2v_+(t)$. It is worth to mention that unlike the input impedance

($Z(f)$ in frequency or $z(t)$ in time), $\hat{z}(t)$ depends on $v_+(t)$ and hence is not a system property. On the other hand

$$Z(f) = Z_0 \frac{1 + S_{11}(f)}{1 - S_{11}(f)},$$

and does not depend on the input. In general, the Fourier (or its inverse) transform of a fraction is not the ratio of the Fourier (or inverse) transforms of the numerator and denominator. Therefore, $\hat{z}(t) \neq z(t)$.

Usually the source used to calculate $\hat{z}(t)$ is a step function (i.e., $v_+(t) = u(t)$). In this case,

$$\hat{z}(\infty) = Z(0),$$

or in another words, the instantaneous impedance settles at the DC value, when the input is a unit step.

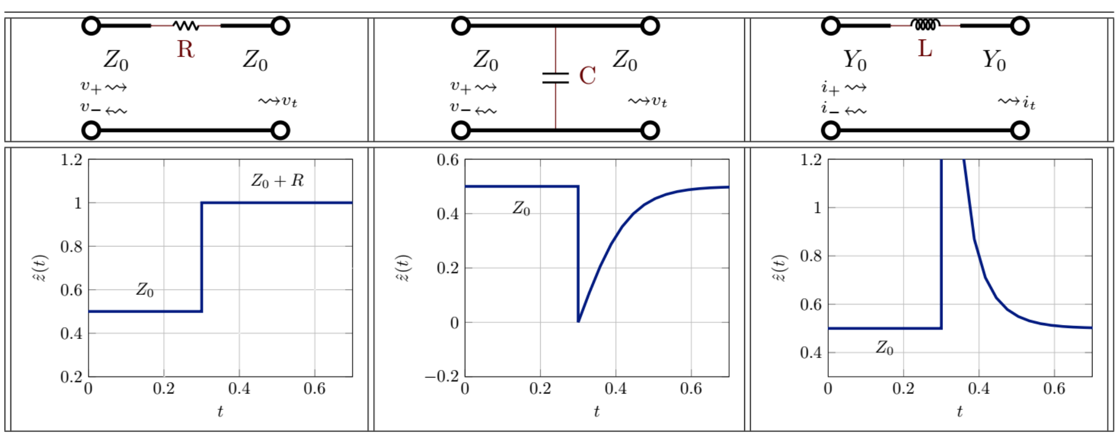

Figure 7 Instantaneous Impedance (TDR) of three simple discontinuities.

To demonstrate how $\hat{z}(t)$ can be calculated for simple discontinuities, we use the expressions for $s_{11}(t)$ reported in Figure 5 and the fact that for a unit step input

$$v_+(t) * s_{11}(t) = \int_0^t s_{11}(t')dt'$$

to calculate $\hat{z}(t)$ using the expression for the instantaneous impedance. Figure 7 shows $\hat{z}(t)$ for three different discontinuities.

Calculating $\hat{z}(t)$ from the frequency domain Data

In a real world setting, an expression for $s_{mn}(t)$ will not be available. In this case, $s_{mn}(t)$ is calculated from its frequency counterpart $S_{mn}(f)$ via the use of inverse Fourier transform. It is worth to briefly highlight the effect of the limited frequency domain data on the computed $s_{mn}(t)$. The first limitation results from the absence of the DC point. As has been already discussed, the DC point determines the steady state value of $\hat{z}(t)$.

The hardware resources used to measure or simulate the S parameters will usually limit the maximum frequency f_m. This implies that the time domain data is smeared by a sinc window of a width $2/f_m$. Hence, discontinuities that are sufficiently close (the distance between them is of the order of c/f_m) will not be resolved by the TDR. Furthermore, small f_m values may lead to a non-causal behavior.

The third and last limitation is due to the resolution or step frequency f_r. If f_r is not small enough, aliasing my occur leading to meaningless $s_{mn}(t)$.

2X Automatic Fixture Extraction

Automatic Fixture Extraction (AFR) is a technique that intelligently exploits both the time and frequency representations to enable the extraction of fixture network parameters [1] [7]. Consider the setup shown in Figure 8. Let us assume that the return loss of a DUT needs to be measured. The DUT, usually a chip, is soldered to a PCB trace. The other end of the trace is connected to the VNA via a suitable coaxial interface. It is assumed that the VNA is calibrated at the coaxial interface. The structure between the coax interface and the DUT is known as the fixture. It can be a simple single or differential trace or may include lumped elements needed to bias and place DUT in the appropriate setting. The presence of the fixture modifies the measured return loss. Instead of measuring the DUT return loss Γ_{DUT}, we end up measuring Γ_{DUT} combined with the fixture S parameters i.e.,

$$\Gamma_{meas} = S_{11} + \frac{S_{21}^2 \Gamma_{DUT}}{1 - S_{22}\Gamma_{DUT}}.$$

Therefore in order to determine Γ_{DUT}, the fixture S parameters $S_{11}, S_{21}$and S_{22} must be calculated at each frequency of interest. This process is known as fixture extraction.

There are different fixture extraction algorithms. The simplest and most direct one is the SOL (Short, Open, Load) method. In this method, three separate fixtures are fabricated such that three different terminations, usually an open, a short and a reference termination (load) replace the DUT. The three independent measurements allow the extraction of the unknown quantities (S_{11}, S_{21} and S_{22}).

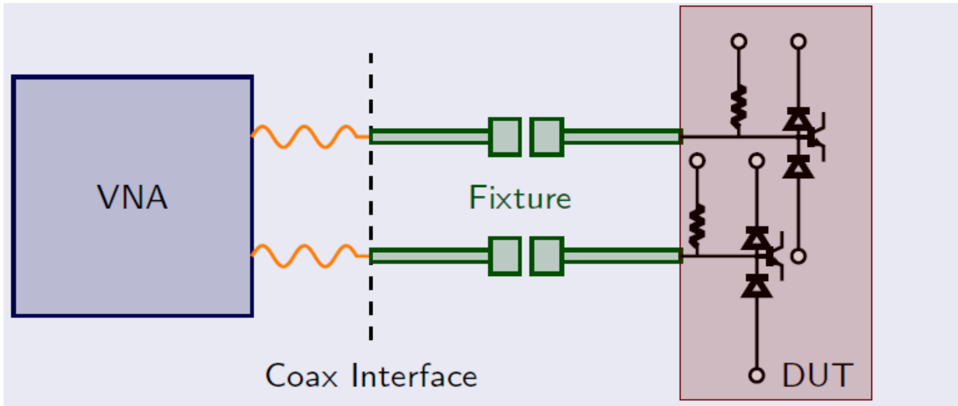

Figure 8 A typical setup used to measure the RL of a DUT.

Another approach that will be discussed in more detail is 2X AFR. A test coupon, which is the fixture cascaded with its mirror image is fabricated (Figure 9). The coupon is symmetric at the mid plane, where the DUT is supposed to be mounted in the original fixture. A full two port (or four port if the fixture is differential) measurement of the test coupon is performed using a VNA that has been calibrated at the coaxial interfaces. Hereafter, the S parameters of the original fixture (the 1X structure) and those of the 2X coupon will be denoted by a 1X and 2X superscripts, respectively.

Using basic circuit theory, the 1X S parameters S_{11}^{1X},S_{21}^{1X} and S_{22}^{1X}can be determined from the measured 2X ones S_{11}^{2X},S_{21}^{2X} and S_{22}^{2X}. However, their relations are complex and finding the unknown parameters can be quite challenging. As will be seen, a combination of the time and frequency domain pictures provides an elegant procedure that decouples the different parameters and permits a direct calculation.

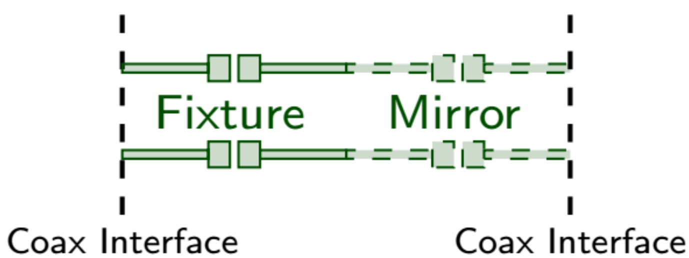

Figure 9 2X structure used to extract the fixture S parameters.

Thinking in terms of the time domain and noting that the 2X structure is symmetric, one can conclude that $s_{11}^{1X}(t)$ is equal to $s_{11}^{2X}(t)$ up to the middle plane. Furthermore, τ_d is the duration of the round trip between the applied impulse and the instant when the reflected wave from the mid plane is observed. Again, due to the structural symmetry, this is equal to the instant when $s_{21}^{2X}(t)$ appears at the output of the 2X structure. Therefore, the first step in the extraction process is to convert $S_{21}^{2X}(f)$ to the time domain and compute the time τ_d at which $s_{21}^{2X}(t)$ starts to appear as shown in Figure 10.

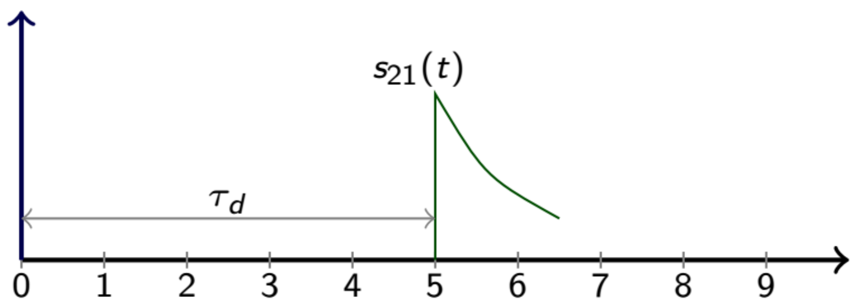

Figure 10 The insertion loss of the 2X structure as it appears in the time domain.

The next step is to calculate $s_{11}^{2X}(t)$ and set $s_{11}^{1X}(t)$ to be

$$s_{11}^{1X}(t) = s_{11}^{2X}(t) \times [1 - u(t - \tau_d)].$$

Note that $s_{11}^{1X}(t)$ calculated above is the reflection coefficient when the termination is the impedance z_{end} at the end-point where the DUT is mounted. Hence, $s_{11}^{1X}(t)$ and consequently $S_{11}^{1X}(f)$ is the reflection coefficient when the DUT side is terminated by z_{end}. Once $S_{11}^{1X}(f)$ is determined, the two other parameters are computed using the frequency domain representation, i.e.

$$S_{22}^{1X}(f) = \frac{S_{11}^{2X}(f) - S_{11}^{1X}(f)}{S_{21}^{2X}}$$

and

$$S_{21}^{1X}(f) = \pm\sqrt{S_{21}^{2X}(f)[1 - (S_{22}^{1X})^2]},$$

where the right sign of $S_{21}^{1X}(f)$ is resolved by enforcing the phase to be a decreasing function of frequency. So far, the extracted parameters S_{11}^{1X}, S_{21}^{1X} and S_{22}^{1X} are normalized to z_{end} at the DUT side. The value of z_{end} can be computed from the instantaneous impedance value $\hat{z}(\tau_d)$. Impedance re-normalization is performed to reference S parameters to the nominal impedance (usually 50 Ω) as Figure 11demonstrates.

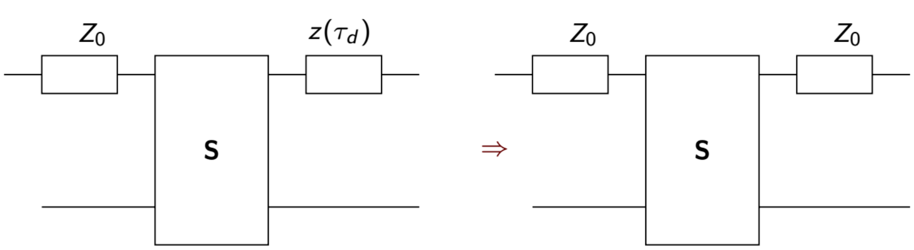

Figure 11 Impedance renormalization.

Up to this point, the procedure was limited to two ports. For high speed circuits, differential (2 Inputs, 2 Outputs) fixtures are ubiquitous. Fortunately, the 2X AFR process described above can be adapted to differential fixtures as shown in Figure 12.

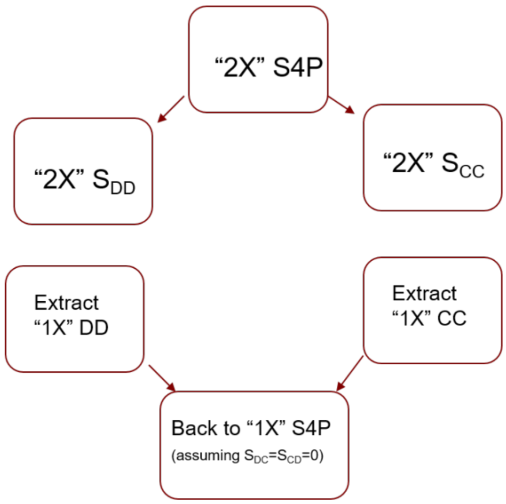

Figure 12 Treating a four port network as two separate two port networks: differential and common mode.

The four port 2X measurement (S4P) is converted to two differential and common mode parameters (S2P) such that

$$S_{DD} = \frac{1}{2} P^t S P$$

and

$$S_{CC} = \frac{1}{2} Q^t S Q,$$

where

$$P = \begin{bmatrix} 1 & 0 \\ -1 & 0 \\ 0 & 1 \\ 0 & -1 \end{bmatrix}$$

and

$$Q = \begin{bmatrix} 1 & 0 \\ 1 & 0 \\ 0 & 1 \\ 0 & 1 \end{bmatrix}.$$

The differential and common mode S parameters S_{DD}^{2X} and S_{CC}^{2X} are two ports and the procedure described in the current paper can be applied to both of them to extract S_{DD}^{1X} and S_{CC}^{1X} that are combined back to form the 1X four port S parameter as Figure 12 shows.

An example that demonstrates the use of the 2X AFR method is shown in Figure 13. Here a circuit model is built from different transmission line sections and lumped elements. The ABCD parameters of the fixture are computed over the frequency range of interest by multiplying the ABCD matrices of each segment. Additionally, the 2X ABCD parameters are produced from the fixture parameters. Both parameters are then converted to four port S parameters.

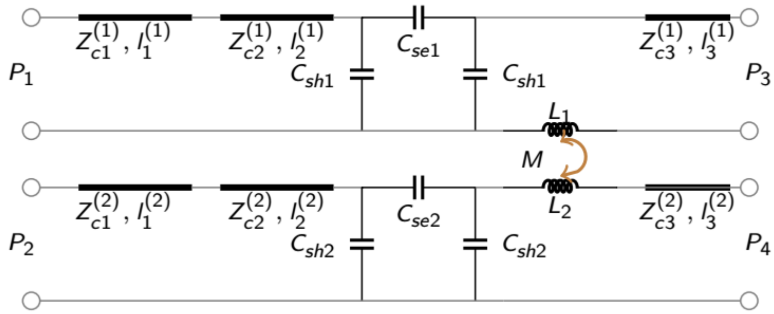

Figure 13 A circuit model of a typical differential interconnect that consists of different transmission line sections and lumped elements. The coupling between the pair is modeled using the mutual inductance M.

The 2X AFR procedure is applied to the 2X S parameters. The extracted 1X S parameters are shown in Figure 14 where they are compared to the ones calculated analytically.

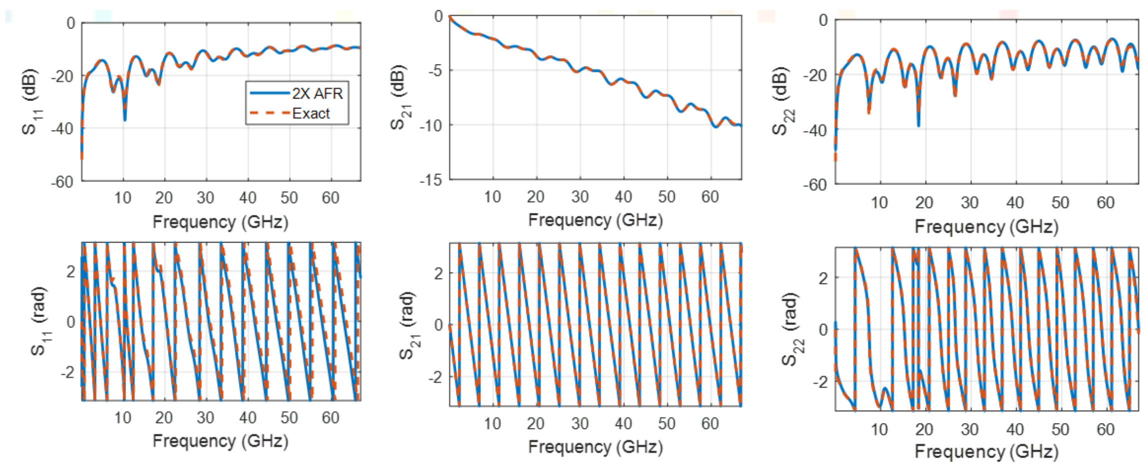

Figure 14 Differential S parameters of 1X fixture.

Figure 14 demonstrates the accuracy of the 2X AFR extraction procedure.

Conclusion

S parameters are re-introduced in the current paper to emphasize the fact that they represent the physical properties of an electrical network as probed at its ports. The approach taken here distinguishes between a property and its representation, whether it is in time or frequency. The S parameters in the time domain are calculated for simple discontinuities to motivate engineers to reason and think in terms of the time domain. Additionally, the relation between S parameters and TDR is discussed and the instantaneous impedance for simple discontinuities are derived. The effect of limited frequency domain data on the instantaneous impedance is briefly presented. Furthermore, we give an overview of how the time and frequency pictures furnish an elegant procedure to extract the network parameters of a fixture. Finally, the extraction procedure is applied to a fixture and results are compared with the analytical calculations.

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